

**Report of Investigations 7937**

# **Vibrations From Underground Blasting**

**By James J. Snodgrass and David E. Siskind  
Twin Cities Mining Research Center, Twin Cities, Minn.**

**US Department of Interior  
Office of Surface Mining  
Reclamation and Enforcement**



**Kenneth K. Eltschlager**  
Mining/Blasting Engineer  
3 Parkway Center  
Pittsburgh, PA 15220

Phone 412.937.2169  
Fax 412.937.3012  
[Keltschl@osmre.gov](mailto:Keltschl@osmre.gov)



**UNITED STATES DEPARTMENT OF THE INTERIOR  
Rogers C. B. Morton, Secretary**

**BUREAU OF MINES  
Thomas V. Falkie, Director**

This publication has been cataloged as follows:

**Snodgrass, James J**

Vibrations from underground blasting, by James J. Snodgrass and David E. Siskind. [Washington] U.S. Bureau of Mines [1974]

31 p. illus., tables. (U.S. Bureau of Mines. Report of investigations 7937)

Includes bibliography.

1. Blast effect. 2. Mining engineering. I. U.S. Bureau of Mines. II. Siskind, David E., jt. auth. III. Title. (Series)

TN23.U7 no. 7937 622.06173

U.S. Dept. of the Int. Library

## CONTENTS

	<u>Page</u>
Abstract.....	1
Introduction.....	1
Test sites.....	5
White Pine.....	5
Shullsburg.....	7
NORAD.....	7
Pilot Knob.....	8
Measurements and analysis.....	9
Acceleration measurements.....	10
White Pine.....	10
Pilot Knob.....	13
Velocity measurements.....	14
Shullsburg.....	14
NORAD.....	18
Conversion of acceleration to velocity.....	18
Numerical methods.....	20
Electronic integration.....	20
Simple harmonic motion.....	22
Scaling factors.....	23
Propagation equations.....	25
Discussion of results.....	27
Conclusions.....	30
References.....	31

## ILLUSTRATIONS

1. Stratigraphic column and mine headings, White Pine ore body.....	4
2. Thin section, parting shale of White Pine ore body.....	7
3. Stratigraphic column and test heading, Shullsburg mine.....	8
4. White Pine test headings 1-2.....	9
5. White Pine test headings 3-4.....	12
6. White Pine test heading 5.....	13
7. White Pine full-column round.....	15
8. White Pine parting shale round.....	15
9. Pilot Knob test heading.....	16
10. Typical Shullsburg production round.....	17
11. Map of Shullsburg test heading.....	19
12. Typical NORAD blast round.....	20
13. Plan and profile views of NORAD exploratory drift.....	21
14. Comparison of regression lines representing directly recorded and derived velocities.....	24
15. Least-squares regression lines of Pilot Knob data after (a) simple harmonic conversion and (b) electronic integration.....	24
16. Least-squares regression lines of square-root-scaled velocity data..	26
17. Least-squares regression lines of square-root-scaled acceleration data.....	26
18. Least-squares regression lines of cube-root-scaled velocity data....	27
19. White Pine velocity data.....	28

Report of Investigations 7937

# Vibrations From Underground Blasting

By James J. Snodgrass and David E. Siskind  
Twin Cities Mining Research Center, Twin Cities, Minn.

**US Department of Interior**  
**Office of Surface Mining**  
**Reclamation and Enforcement**



**Kenneth K. Eltschlager**  
Mining/Blasting Engineer  
3 Parkway Center  
Pittsburgh, PA 15220

Phone 412.937.2169  
Fax 412.937.3012  
[Keltschl@osmre.gov](mailto:Keltschl@osmre.gov)



**UNITED STATES DEPARTMENT OF THE INTERIOR**  
**Rogers C. B. Morton, Secretary**

**BUREAU OF MINES**  
**Thomas V. Falkie, Director**

This publication has been cataloged as follows:

**Snodgrass, James J**

Vibrations from underground blasting, by James J. Snodgrass and David E. Siskind. [Washington] U.S. Bureau of Mines [1974]

31 p. illus., tables. (U.S. Bureau of Mines. Report of investigations 7937)

Includes bibliography.

1. Blast effect. 2. Mining engineering. I. U.S. Bureau of Mines. II. Siskind, David E., jt. auth. III. Title. (Series)

TN23.U7 no. 7937 622.06173

U.S. Dept. of the Int. Library

## CONTENTS

	<u>Page</u>
Abstract.....	1
Introduction.....	1
Test sites.....	5
White Pine.....	5
Shullsburg.....	7
NORAD.....	7
Pilot Knob.....	8
Measurements and analysis.....	9
Acceleration measurements.....	10
White Pine.....	10
Pilot Knob.....	13
Velocity measurements.....	14
Shullsburg.....	14
NORAD.....	18
Conversion of acceleration to velocity.....	18
Numerical methods.....	20
Electronic integration.....	20
Simple harmonic motion.....	22
Scaling factors.....	23
Propagation equations.....	25
Discussion of results.....	27
Conclusions.....	30
References.....	31

## ILLUSTRATIONS

1. Stratigraphic column and mine headings, White Pine ore body.....	4
2. Thin section, parting shale of White Pine ore body.....	7
3. Stratigraphic column and test heading, Shullsburg mine.....	8
4. White Pine test headings 1-2.....	9
5. White Pine test headings 3-4.....	12
6. White Pine test heading 5.....	13
7. White Pine full-column round.....	15
8. White Pine parting shale round.....	15
9. Pilot Knob test heading.....	16
10. Typical Shullsburg production round.....	17
11. Map of Shullsburg test heading.....	19
12. Typical NORAD blast round.....	20
13. Plan and profile views of NORAD exploratory drift.....	21
14. Comparison of regression lines representing directly recorded and derived velocities.....	24
15. Least-squares regression lines of Pilot Knob data after (a) simple harmonic conversion and (b) electronic integration.....	24
16. Least-squares regression lines of square-root-scaled velocity data..	26
17. Least-squares regression lines of square-root-scaled acceleration data.....	26
18. Least-squares regression lines of cube-root-scaled velocity data....	27
19. White Pine velocity data.....	28

## ILLUSTRATIONS--Continued

	<u>Page</u>
20. Shullsburg velocity data.....	28
21. NORAD velocity data.....	29
22. Pilot Knob velocity data.....	29

## TABLES

1. Physical properties of rock at test sites.....	6
2. White Pine blast data.....	11
3. Pilot Knob blast data.....	14
4. Shullsburg blast data.....	18
5. NORAD blast data.....	18
6. Correlation coefficients of least-squares regression lines for square root and cube root scaling.....	25
7. Values of site constants for square root and cube root scaling.....	25

# VIBRATIONS FROM UNDERGROUND BLASTING

by

James J. Snodgrass<sup>1</sup> and David E. Siskind<sup>1</sup>

---

---

## ABSTRACT

The Bureau of Mines has investigated vibration levels produced by blasting at four underground sites to establish how such factors as type of explosive, delay blasting, charge weight, and geology affect amplitudes of ground motion. A summary of the work is presented and the results of further analysis of the data are discussed. Square root scaling was found applicable to two of the underground sites and could be applied with minor error to all the sites. Comparison of empirical propagation equations in the different rock types indicates that although the site effect is apparent, the combined data may be used as a basis for engineering estimates of vibration amplitudes from subsurface blasting in many different rock types. Recommendations for predicting and minimizing vibration amplitudes from underground blasts are given.

## INTRODUCTION

Effects of vibrations from blasting at quarry sites have been thoroughly investigated by the Bureau of Mines with the goal of establishing a reliable, technological basis to plan and conduct blasting operations that will minimize the number of damage claims and nuisance complaints received by operators of quarries, open pit mines, and surface construction projects. This research has resulted in a comprehensive analysis of parameters affecting blast vibration amplitudes. Guidelines for safe blast design, methods to reduce vibrations without altering the efficiency of a blast round, and empirical means to predict maximum vibration amplitudes have been established that should prove beneficial to both users of explosives and the public.

Nicholls, Johnson, and Duvall (4)<sup>2</sup> recently reported results of the Bureau's 10-year program to evaluate the problem of air and ground vibrations from blasting. In addition to the published work of others, field data from 171 blasts at 26 quarry sites representing numerous rock types were analyzed to establish damage criteria for residential structures and to make recommendations for safe blast design. Some conclusions from the study were

---

<sup>1</sup>Geophysicist.

<sup>2</sup>Underlined numbers in parentheses refer to items in the list of references at the end of this report.

as follows: (1) Damage to residential structures from blasting correlates more closely with the resultant particle velocity of ground motion than with acceleration or displacement; (2) a peak particle velocity of 2.0 in/sec ( $5 \times 10^{-2}$  m/sec)<sup>3</sup> as measured from any of three mutually perpendicular directions in the ground adjacent to a structure is a reasonably safe upper limit to preclude damage to the structure; (3) humans respond uncomfortably to levels of vibration that are lower than those necessary to cause damage to structures; (4) millisecond delay blasting may be used to reduce vibration levels since resultant amplitudes are determined by maximum charge weight per delay, not total charge weight; (5) and dividing the distance from the blast by the square root of the charge weight per delay is an effective method for relating data from different sized blasts to determine an empirical wave propagation equation for a particular site.

Based on these findings, it was recommended that seismic instrumentation be used to determine the propagation characteristics of a site to insure that the safe blasting criterion of 2.0 in/sec peak particle velocity is not exceeded. If instrumentation is not available, a reasonable margin of safety from damage to structures is provided when a scaled distance of 50 ft/lb<sup>1/2</sup> (22.6 m/kg<sup>1/2</sup>) is used. In equation form,

$$D \geq 50W^{1/2}, \quad (1)$$

or conversely  $W \leq (D/50)^2, \quad (2)$

where D is the minimum distance in feet from the blast and W is the maximum charge weight in pounds per delay period. Thus, for example, at a distance of 1,000 ft from a structure, the maximum charge weight per delay that may be detonated with small probability of damage would be 400 lbs. In situations where nuisance complaints are to be minimized, stricter limits may be required.

<sup>3</sup>The prime units in the text, tables, and illustrations of this publication are the U.S. customary units. Where appropriate, the approximate equivalents in the International System of Units (SI) are included in accordance with the rules for introducing modernized metric units established in the National Bureau of Standards ASTM Metric Practice Guide, E380-70. In accordance with the SI convention, a space rather than a comma is used to separate the digits in a metric number such as 15 000. The U.S. customary numbers used throughout the report include commas, where necessary, to separate the digits. The period is used as a decimal point in both SI and U.S. customary numbers.

#### Abbreviations

U.S. customary units	SI units
in = inch	mm = millimeter
ft = foot	m = meter
lb = pound	kg = kilogram
sec = second	sec = second
mV = millivolts	V = volts
g = unit of gravity	

Four subsequent Bureau investigations of blast vibrations involved measurements on the roof and walls of underground mines. In the first study, Olson, Dick, Condon, and Fogelson (5) monitored particle accelerations from small-scale and production blasts in the shale formation of the White Pine Copper mine, White Pine, Mich. As in the quarry studies, square root scaling was found to be an effective technique to group data from different sized blasts. Maximum vibration amplitudes, produced by the zero-delay portion of each blast, could be reduced by decreasing the amount of explosive in the zero-delay period. Also, the use of ammonium nitrate-fuel oil (AN-FO) blasting agent produced smaller vibration levels than did dynamite. Vibration differences among mine headings and explosive types were not major factors in estimating maximum vibration amplitudes with an empirically derived propagation law.

The second study by Olson, Dick, Fogelson, and Fletcher (6) was conducted in the limestone-dolomite roof rock of the Shullsburg zinc-lead mine Shullsburg, Wis. Both particle acceleration and velocity measurements were obtained for single-hole shots and production rounds. Again, the zero-delay portion of production rounds produced the maximum vibration amplitudes, even though the charge weight of each subsequent delay period sometimes exceeded that of the zero delay. In contrast to the previous studies, cube root scaling appeared to be more appropriate for the Shullsburg site, suggesting that the geometry of a blast round and the configuration of the opening may have a pronounced influence on the scaling factor. Square root scaling was used, however, to compare the Shullsburg and White Pine data, which were in good agreement.

In the third study, Olson, Fogelson, Dick, and Hendrickson (7) obtained particle velocity measurements in the walls of a tunnel blasted in the Pikes Peak Granite of the North American Air Defense Command (NORAD) Complex near Colorado Springs, Colo. Since dimensional analysis supports cube root scaling and because the nature of the excavation at NORAD was significantly different from either White Pine or Shullsburg, a statistical analysis was performed on the NORAD data to determine whether square root or cube root scaling was more appropriate. The cube root factor was found more effective in removing the effect of various charge weights. An empirical propagation equation derived from the cube-root-scaled data was used successfully to predict maximum vibration amplitudes during subsequent expansion of the excavation.

The Pilot Knob Pellet Co.'s underground iron ore mine near Pilot Knob, Mo., provided the opportunity to study vibrations in yet another rock type. Siskind (12) recorded particle accelerations in the roof rock of a development drift excavated in the massive, competent magnetite ore body. No data from production blasting were obtained; however, a designed experiment using a range of charge weights in single-hole blasts allowed determination of a best scaling factor and a propagation equation for the site. Square root scaling was a good approximation of the statistically determined best fit value of 0.55. Vibration measurements from different types of explosives were in agreement with the findings of Olson, Dick, Condon, and Fogelson (5) that when other factors allow, AN-FO may be used to reduce vibration amplitudes.

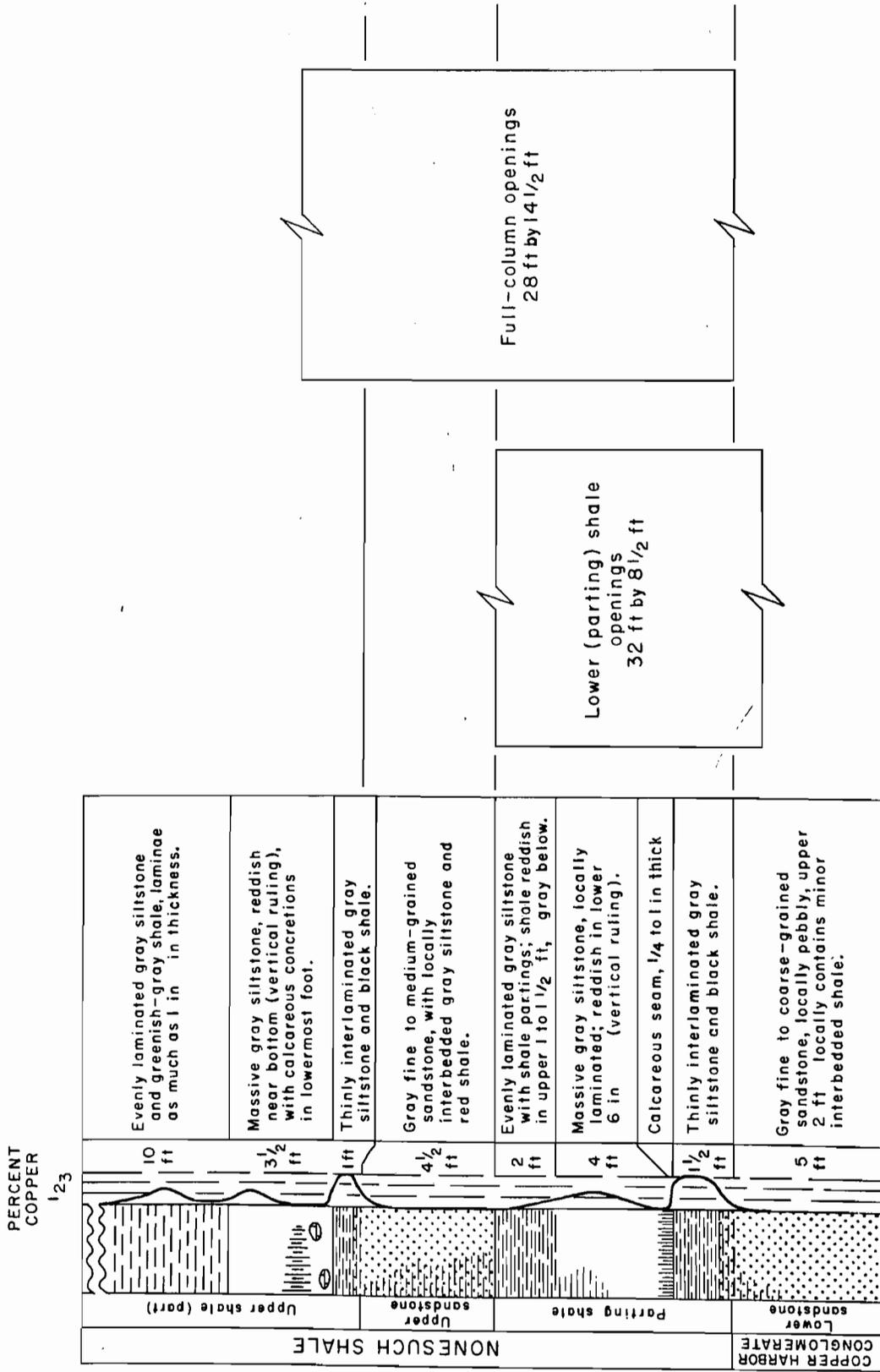


FIGURE 1. - Stratigraphic column and mine headings, White Pine ore body.

Of primary importance in underground excavation is the requirement to maintain rock stability and prevent rockfalls or damage to support structures. The high-intensity elastic waves induced by production blasting and the adjustment of rock due to stresses associated with the opening itself create conditions that affect rock competency outside the excavation boundary. Scott, Lee, Carroll, and Robinson (10) observed these effects and demonstrated the potential for geophysical techniques to predict engineering and economic aspects of tunnel excavation. Seismic refraction and electrical resistivity measurements indicated an altered zone commonly extending 10 ft (3.0 m) or more into the walls of a pilot bore driven with explosives in granite. Physical property changes were attributed to (1) blast damage in the first few feet of the altered zone and (2) stress redistribution in the deeper layer. Increasing seismic amplitudes were correlated with increasing fracture spacing observed along the walls of the pilot bore, and increasing resistivity values were correlated with decreasing support requirements, suggesting methods to define unstable zones where rockfalls might be expected and additional support required.

Although the problems associated with the use of explosives underground differ from those encountered in surface blasting operations, simultaneous urban expansion and subsurface excavation near urban areas for transportation systems, utilities, and storage emphasize the need to understand and control earth vibration caused by blasting. Means for predicting vibration levels in advance of blasting operations and methods to reduce maximum amplitudes while maintaining rock fragmentation capability will lead to more efficient design of underground blast rounds and contribute to a safer working environment. This report summarizes the studies of underground blasting and combines the data to estimate expected vibration levels over a wide range of geologic conditions and mining configurations.

#### TEST SITES

Vibrations were measured in underground mines representing four different rock types: Shale, limestone, granite, and magnetite ore. Table 1 summarizes the physical properties of the mine rock as measured in the individual studies.

#### White Pine

The room-and-pillar White Pine Copper mine is located 4 miles south of Lake Superior in Ontonagan County, Upper Peninsula of Michigan (5). The ore body consists primarily of the lower 20 to 25 ft (6.1 to 7.6 m) of the Nonesuch Shale and the upper few feet of the Copper Harbor Conglomerate. The presence of numerous faults, joints, and compositional changes influenced the level and rate of attenuation of the blast vibrations in the instrumented headings. Detailed mappings of the test headings showing the local geologic structure are given in reference 5. Figure 1 shows the stratigraphic column of the White Pine ore body and the parting shale and full-column headings. Petrographic study of thin sections from the parting shale indicated that it is a massive to thinly laminated gray siltstone consisting of thin carbonaceous laminae separated by thicker laminae of dominantly silt-sized quartz grains (13). The massive siltstone is composed of uniformly interspersed silt-size particles of quartz, micaceous minerals, and carbonaceous material. Discrete areas of calcite were found on many of the thin sections (fig. 2).

TABLE 1. - Physical properties of rock at test sites

	White Pine, Mich., Nonesuch Shale		Shullsburg, Wis., Platteville Formation Quimby's Mill Member		NORAD, Colo., Pikes Peak Granite		Pilot Knob, Mo., Bottom ore, magnetite	
	Upper sandstone		Parting shale		U.S. customary		U.S. customary	
	U.S. customary	SI units	U.S. customary	SI units	U.S. customary	SI units	U.S. customary	SI units
Specific gravity.....	2.59	2.73	2.59	2.59	2.59	2.59	3.90	3.90
Compressive strength lb/in <sup>2</sup> (N/m <sup>2</sup> )..	21,550	148 x 10 <sup>6</sup>	24,150	167 x 10 <sup>6</sup>	17,500	121 x 10 <sup>6</sup>	42,000- 58,000	290- 400 x 10 <sup>6</sup>
Tensile strength lb/in <sup>2</sup> (N/m <sup>2</sup> )..	1,690	11.7 x 10 <sup>6</sup>	1,330	9.17 x 10 <sup>6</sup>	757	5.22 x 10 <sup>6</sup>	-	-
Brazilian tensile strength lb/in <sup>2</sup> (N/m <sup>2</sup> )..	-	-	-	-	-	-	2,300- 2,800	15.9- 19.3 x 10 <sup>6</sup>
Young's modulus (static) lb/in <sup>2</sup> (N/m <sup>2</sup> )..	6.6 x 10 <sup>6</sup>	46 x 10 <sup>9</sup>	7.1 x 10 <sup>6</sup>	49 x 10 <sup>9</sup>	5.96 x 10 <sup>6</sup>	41.1 x 10 <sup>9</sup>	5.04 x 10 <sup>6</sup>	34.7 x 10 <sup>9</sup>
Young's modulus (dynamic) lb/in <sup>2</sup> (N/m <sup>2</sup> )..	-	-	-	-	-	-	4.34 x 10 <sup>6</sup>	29.9 x 10 <sup>9</sup>
Shear modulus (dynamic). Poisson's ratio (dynamic).....	-	-	-	-	-	-	2.18 x 10 <sup>6</sup>	15.0 x 10 <sup>9</sup>
Longitudinal velocity ft/sec (m/sec)..	16,800	5 130	17,300	5 280	18,400	5 620	13,100	4 010
Torsional velocity ft/sec (m/sec)..	10,300	3 130	10,100	3 080	9,910	3 020	7,740	2 360
Bar velocity ft/sec (m/sec)..	15,800	4 820	15,700	4 790	15,000	4 580	10,900	3 320

<sup>1</sup> Properties determined by the University of Missouri-Rolla for the Pilot Knob Pellet Co formation known as the "lower ore." All other properties were determined at the Twin Cities Mining Research Center.

Shullsburg

The Shullsburg mine is a room-and-random pillar zinc-lead mine located in Lafayette County in the extreme southwest corner of Wisconsin (6). The ore body is of the pitch-and-flat type where the pitches are mineralized inclined fractures crossing the bedding planes at odd angles and the flats are mineralized bedding plane fractures. The test heading was comprised of the top part of the McGregor Limestone and the bottom part of the Quimby's Mill Members of the Platteville Formation (fig. 3). The Quimby's Mill roof rock at the test site appeared vuggy, and the roof surface contained undulations; however, unlike

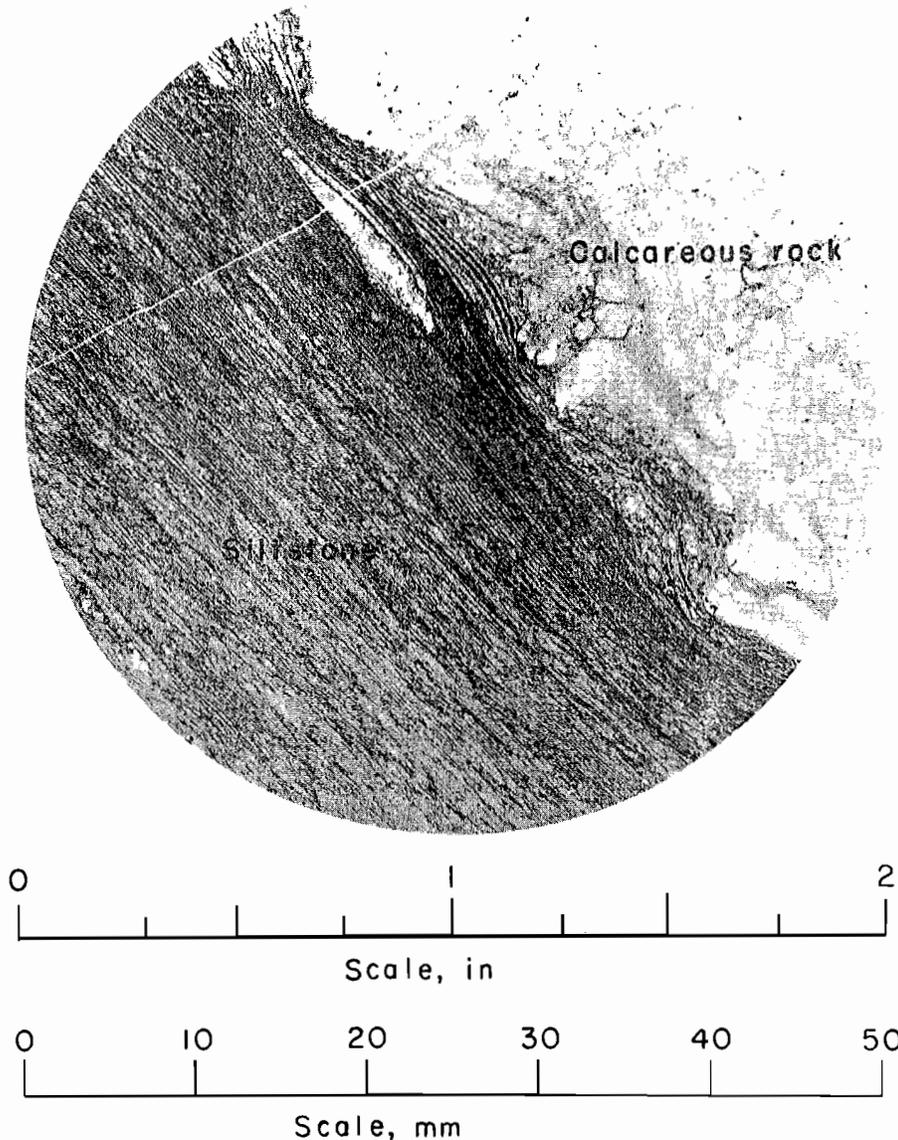


FIGURE 2. - Thin section, parting shale of White Pine ore body.

the White Pine site, there were no major fractures and only a few spall areas observed. The test heading is further described in a report by Olson, Dick, Fogelson, and Fletcher (6).

NORAD

The NORAD site is an underground complex housing the Air Force Combat Operation Center located at Cheyenne Mountain, 5 miles south of Colorado Springs, Colo. (7).

Formations	Members	Miners' terms	Columnar section	Depth below surface, first value in meters, second value in feet.
Galena Dolomite	Cherty lower unit	Drob	Dolomite	---
Decorah Formation	Ion Dolomite	Gray	Dolomite	---
		Blue	Dolomite and limestone	91.7 (301)
	Guttenburg Limestone	Oil rock	Limestone and dolomite	94.2 (309)
Platteville Formation	Quimby's Mill	Glass rock	Limestone and dolomite	96.8 (317.5)
	McGregor Limestone	Trenton	Limestone and dolomite	101.5 (333)

Test heading

FIGURE 3. - Stratigraphic column and test heading, Shullsburg mine.

The rock is classified as Pikes Peak Granite of Precambrian age. The test site was in an exploratory drift in an area of new construction, and detailed geologic mapping by the Corps of Engineers found the rock to be a coarse-grained biotite granite intruded by a fine-to-medium-grained granite. The rock is essentially fresh and unaltered except for some thin zones, with fractures in the rock mass being moderately to closely spaced. Generally, the physical measurements (table 1) indicate that this section of the Pikes Peak Granite is a less competent rock than would be expected.

#### Pilot Knob

The Pilot Knob magnetite iron mine is located 1 mile east of a town of the same name in Iron County, Mo., 70 miles south of St. Louis (12). The mining method involves a unique combination of room-and-pillar and sublevel stoping techniques. The test heading was in the massive body of ore identified as "lower ore" and described as a medium- to fine-grained magnetite-quartz-feldspar mixture. Ore grade averaged 37 to 39 pct magnetic iron with hematite as a minor constituent. Most notable are the high strength values (table 1).

Stratigraphic columns for this site and the NORAD study are not given because the headings, shots, and measuring gages are all within a single massive rock body.

## MEASUREMENTS AND ANALYSIS

Particle velocities and accelerations were measured underground at the four sites. The experiments were similar in design, although blast and gage geometries varied somewhat from site to site. Shots fired in an advancing working face were detected with a linear array of gages mounted on the drift roof at White Pine, Shullsburg, and Pilot Knob, and on the wall near the floor at NORAD. At each site except NORAD, gages were logarithmically spaced; most of them were placed near the shot. The surface vibration work yielded

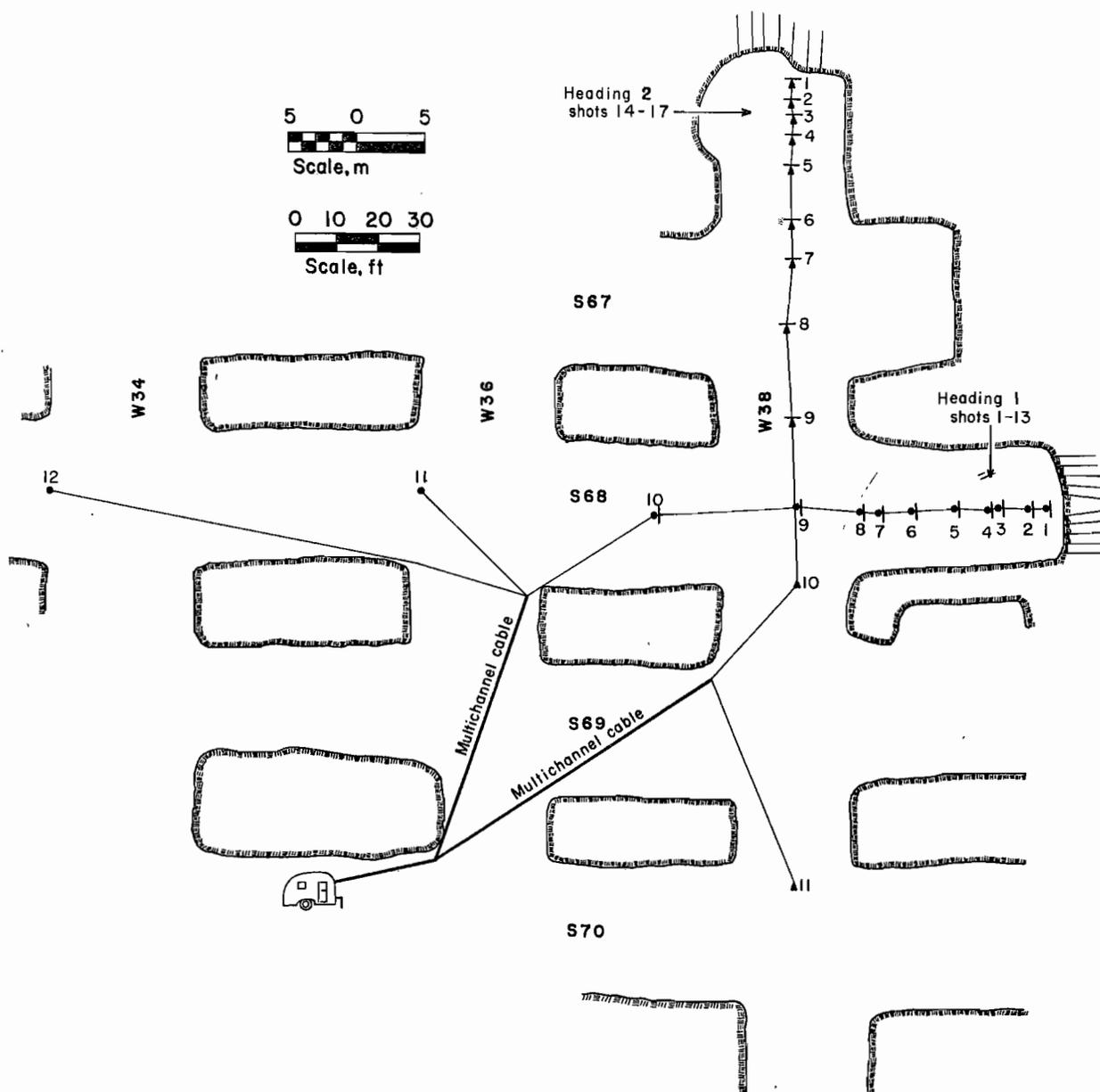


FIGURE 4. - White Pine test headings 1-2.

exponential propagation equations that graphed as straight lines on log-log paper and similar results were expected from the underground studies. With the logarithmic gage spacing, data points are more evenly distributed when plotted on log-log paper.

Measurements were recorded on magnetic tape at high speed (60 in/sec) and played back at low speed (1-7/8 in/sec) through appropriate amplifiers to a direct-writing oscillograph giving high-resolution records of ground motion as a function of time ranging in frequency from 60 Hz to over 10 kHz. Scaling of the distances by the charge weights was examined, and final results were propagation laws relating levels of vibration to charge weights and shot-to-gage distances for each site.

### Acceleration Measurements

Particle accelerations were measured in three mines: White Pine, Shullsburg, and Pilot Knob. The Shullsburg study involved acceleration measurements from a single charge size for comparison with the White Pine work; however, an extensive particle velocity study was made at this site and is discussed in a later section. The White Pine and Pilot Knob studies involved measurements over a range of 0.04 to 870 g (3.9 to 8526 m/sec<sup>2</sup>) and 60 to 20,000 Hz, far beyond the capability of particle velocity instrumentation. The recording system was designed to have a flat response from 2 to 20,000 Hz ( $\pm 10$  pct), exclusive of the gages. Near the blast, relatively insensitive accelerometers were used having bandwidths of 2 to 15,000 Hz ( $\pm 10$  pct); further from the blast area, more sensitive gages (2 to 6,000 Hz  $\pm 5$  pct) were employed. For comparison with the extensive surface vibration work and the other underground measurements, some of the acceleration records have been converted to particle velocity.

#### White Pine

The White Pine study involved two distinct phases and a total of five test headings. Phase 1 was conducted in parting shale openings with gages mounted on the base of the upper sandstone roof in headings 1-2 (fig. 4). In heading 1, vibrations were recorded from a series of 13 shots ranging from 1/4 stick to 4 sticks of 1-1/4- by 16-in, 60 pct weight-strength ammonia dynamite. The 1-stick shot size (0.91 lb, 0.41 kg) was selected as a standard for comparisons between headings, and in nearby heading 2, four single-shot rounds were fired. Table 2 summarizes the shot data. This initial work involved experimentation on gage mounting and protection techniques. Two methods of gage attachment were used: (1) Mounting accelerometers on plexiglass cylinders epoxied into holes in the mine roof and (2) direct epoxy mounting. The plexiglass method produced resonance problems and was later discontinued (5). Stemming consisted of paper cartridges filled with fine-grained sand that were tamped into place behind the charge.

TABLE 2. - White Pine blast data

Shot	Type of explosive charge	Number of sticks	Charge size weight		Heading
			Lb	Kg	
1-3.....	Dynamite.....	1/4	0.23	0.10	1
4-6.....	.....do.....	1/2	.45	.20	1
7-9.....	.....do.....	1	.91	.41	1
10-12.....	.....do.....	2	1.84	.83	1
13.....	.....do.....	4	3.64	1.65	1
14-17.....	.....do.....	1	.91	.41	2
18-21.....	.....do.....	1	.91	.41	3
22.....	AN-FO production...	NAP	39	17.7	3
23-25.....	Dynamite.....	1	.91	.41	4
26-29.....	AN-FO production...	NAP	39	17.7	4
30-33.....	Dynamite.....	1	.91	.41	5
34-36, 28..	.....do.....	42	38.2	17.3	5
37.....	AN-FO.....	NAP	39	17.7	5

NAP = Not applicable.

During the second phase of the work, vibrations from production shots in both full-column and parting shale headings were recorded in addition to single-stick shots. Headings 3-4 (fig. 5) were parting shale openings on the fringe of active full-column workings approximately 3,000 ft (920 m) from headings 1-2. Gages were mounted on the upper sandstone roof in parting shale openings (as in headings 1-2); however, the shots were in the full-column workings with blastholes in both the parting shale and the upper sandstone. Heading 5 (fig. 6) involved production and test shots in a parting shale opening. Therefore, the gages were mounted on the base of the upper sandstone, as in the other headings, but in contrast to headings 3-4, blastholes were only in the parting shale. Detailed structural geologies of the five test headings are given by Olson, Dick, Condon, and Fogelson (5).

White Pine production shots consisted of AN-FO detonated with 75 pct weight-strength ammonia gelatin primers in 1-5/8 in-diam holes. Extremely wet headings were blasted with 60 pct weight-strength dynamite. The full-column rounds had 43 holes and were fired with millisecond delays (fig. 7). Total charge weight was approximately 270 lb (123 kg) with from 38 to 39 lb (17.3 to 17.7 kg) in the zero delay. The parting shale rounds had 30 holes totaling 180 lb (82 kg) with the zero-delay portion of 39 lb of AN-FO (or 38.2 lb of dynamite) being the largest charge in the round (fig. 8). The delay caps used at White Pine separated the vibrations from each delay interval and the maximum vibration amplitudes from all production rounds were produced by the zero-delay holes. Blast data are given in table 2.

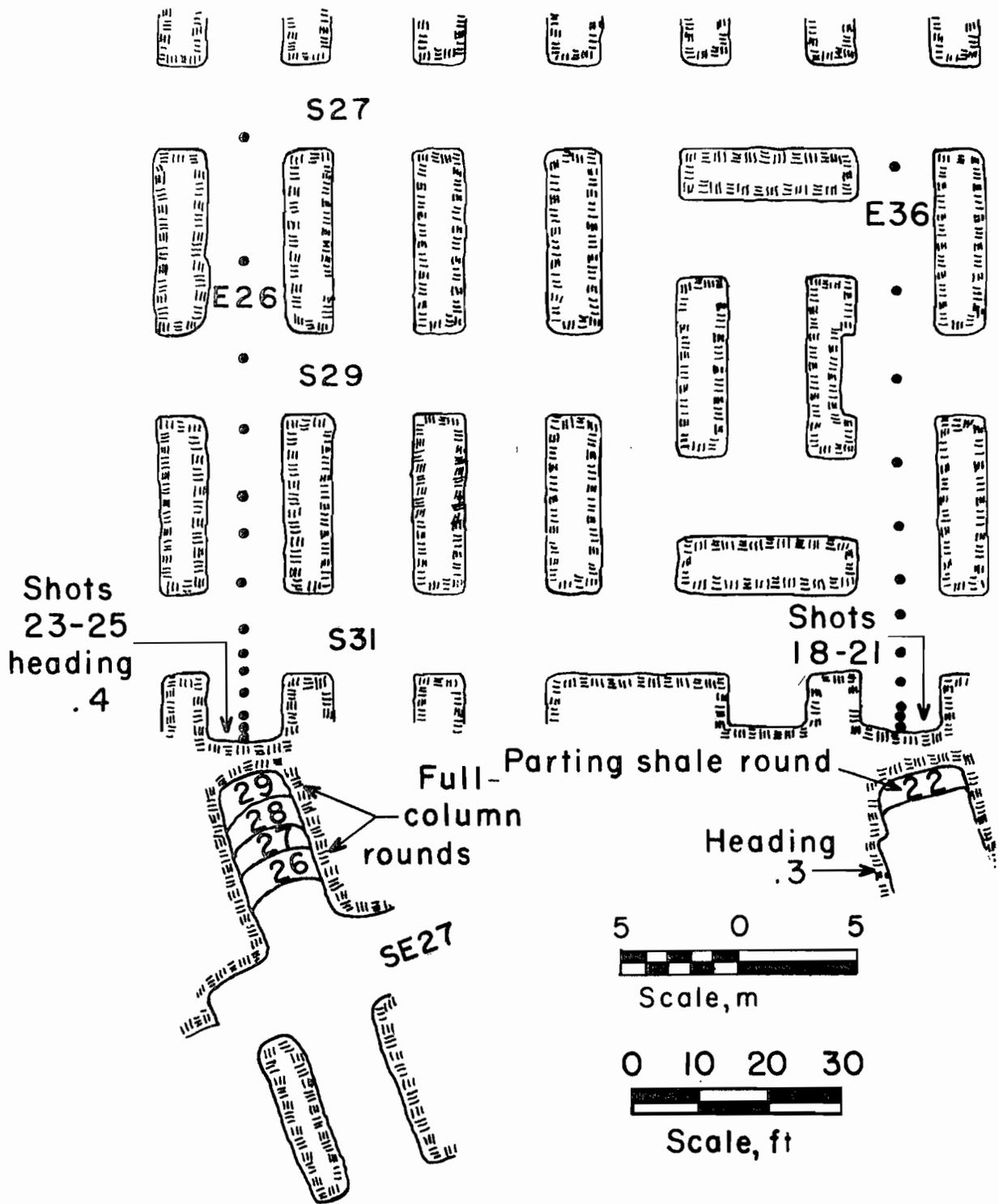


FIGURE 5. - White Pine test headings 3-4.

Pilot Knob

The Pilot Knob study was conducted in a single heading in the magnetite bottom ore. As in the White Pine study, vibrations from a geometric series of charge weights were examined. A total of 17 shots were fired, ranging from 1/3 stick to 10-2/3 sticks of 1-1/2- by 8-in, 75 pct weight strength ammonia gelatin dynamite. The weight of the 1/3 stick of dyanmite for this study (0.228 lb, 0.104 kg) closely corresponded to the 1/3 stick charge for White Pine (0.23 lb, 0.10 kg). The 2-2/3-stick gelatin dynamite charge weight (1.83 lb, 0.833 kg) was selected as a standard size and equivalent weights of AN-FO and slurry (40 pct strength) were shot for explosive comparison. Half-pound cast primers were used to detonate the AN-FO and slurry blasting agents. Blastholes were 2-7/8 in (73 mm) in diameter, and in some cases venting occurred as the fine sand stemming failed to contain all the shots. Table 3 summarizes the Pilot Knob shot data. Gages were mounted on the drift roof using epoxy as at White Pine. Because of wet conditions, some had to be remounted using aluminum plates and 3/8-in anchor bolts. Figure 9 shows the experimental drift including the

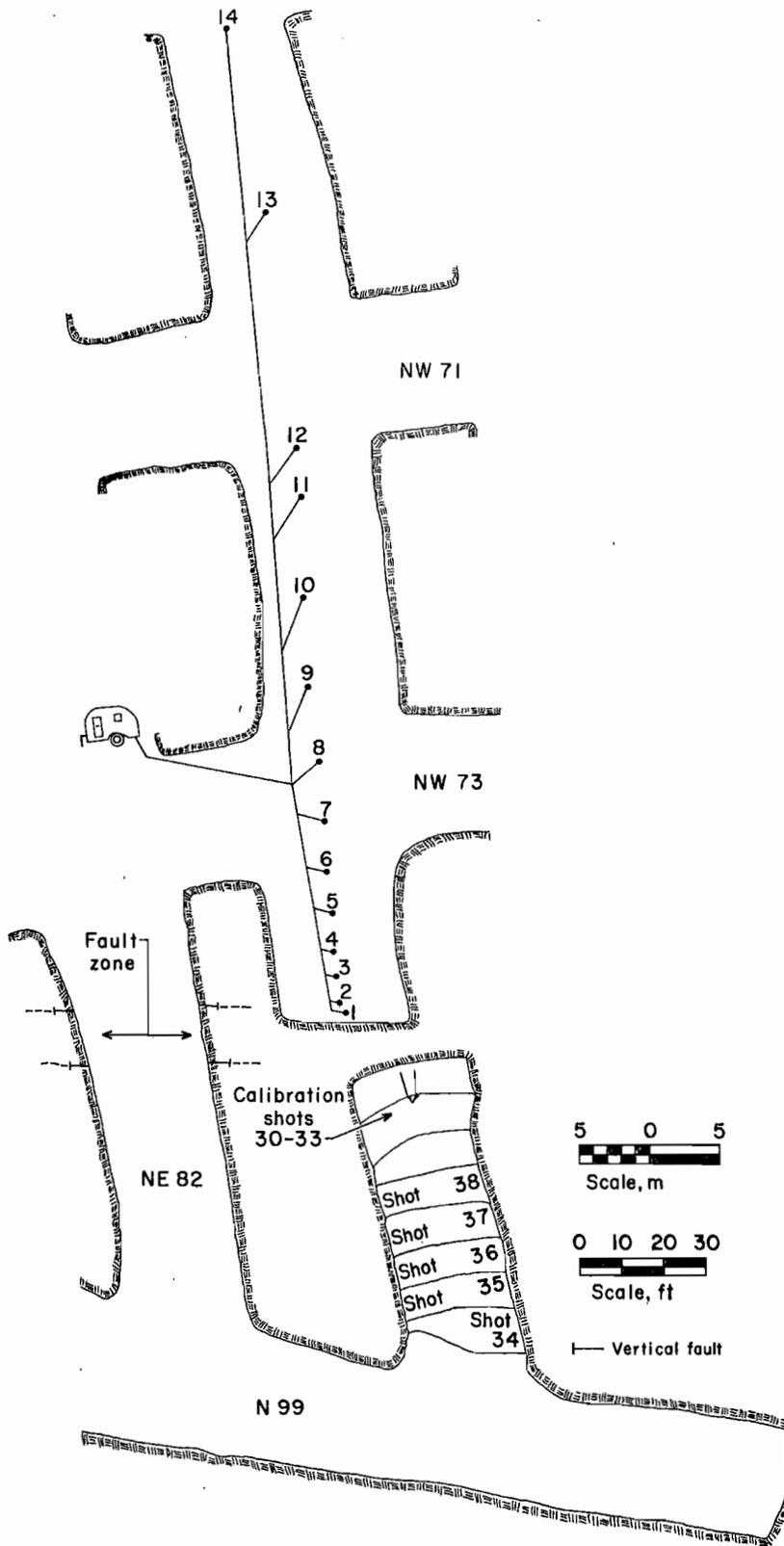


FIGURE 6. - White Pine test heading 5.

12-gage, logarithmically spaced linear array. Recording procedures and equipment were the same as those used in the White Pine study. Siskind (12) describes the details of the shooting procedure, geologic structure, and explosive properties for the Pilot Knob study.

TABLE 3. - Pilot Knob blast data

Shot	Type of explosive charge	Number of sticks	Charge size weight	
			Lb	Kg
1-5, 15.....	Gelatin dynamite.....	1/3	0.228	0.104
6-7, 16.....	.....do.....	2/3	.456	.207
8-9, 17.....	.....do.....	1-1/3	.913	.415
10-11, 18.....	.....do.....	2-2/3	1.83	.833
21.....	.....do.....	5-1/3	3.66	1.66
22.....	.....do.....	10-2/3	7.31	3.32
12, 20.....	AN-FO.....	NAP	1.83	.833
14, 19.....	Slurry.....	NAP	1.83	.833

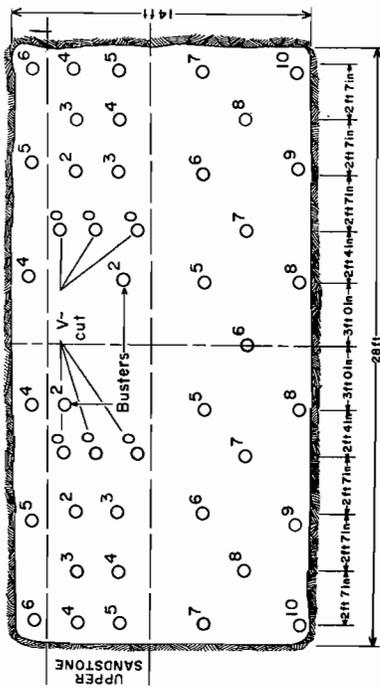
NAP = Not applicable.

#### Velocity Measurements

Particle velocities were measured at the NORAD and Shullsburg sites. The recording system utilized electromagnetic-type velocity gages with a recommended operating range of 10 to 2,000 Hz and a nominal voltage sensitivity of 96.3 mV/in/sec (3.79 V/m/sec) in conjunction with high gain voltage amplifiers (dc to 15,000 Hz  $\pm 3$  pct). Overall system response was therefore suitable for faithful reproduction of a majority of the waveforms, which ranged in frequency from 60 to 2,600 Hz and in amplitude from 0.003 to 4.56 in/sec ( $0.0086 \times 10^{-2}$  to  $11.6 \times 10^{-2}$  m/sec) peak particle velocity. An alternate system employed at NORAD used similar velocity gages but different type voltage amplifiers (2 to 15,000 Hz  $\pm 3$  pct), also giving adequate system response. Signal outputs from these amplifiers were recorded directly using an oscillograph with high performance galvanometers.

#### Shullsburg

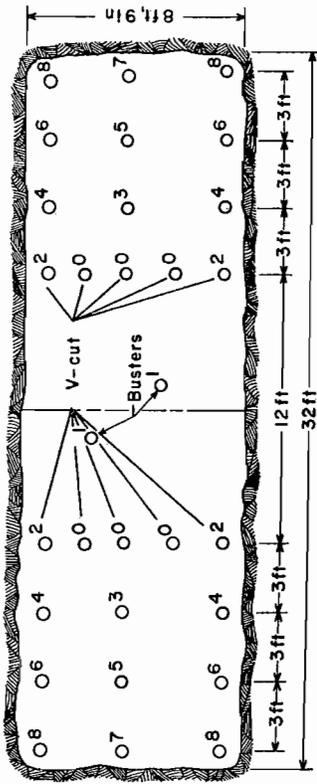
At Shullsburg, vibrations from two types of blast were recorded. Shots 1-8 in the rib at the front of the gage spread each consisted of one 1-1/2-by 12-in (38 by 305 mm) stick of 45 pct semigelatin dynamite weighing 0.92 lb (0.42 kg). Shots were completely stemmed with sand cartridges, confining both the noise and gases produced and allowing return to the area shortly after firing. In addition, nine standard production rounds (fig. 10) were monitored. These employed a V-cut-consisting of two columns of five holes each spaced approximately 6.9 ft (2.1 m) apart at the face and drilled to meet at a depth of 8.2 to 9.8 ft (2.5 to 3.0 m) as shown in figure 9. Two columns of five reliever holes each were collared at about 1.6 ft (0.5 m) from the V-cut holes and angled toward the cut to prevent excessive burdens at the toe. The remainder of the round consisted of additional columns of four holes each drilled about 3.2 ft (1.0 m) from the previous column. Blasthole diameters ranged from 1-3/4 to 2 in (44 to 51 mm). Normal charge per hole



DELAY NUMBER	TIME INTERVAL, msec
0	—
2	50
4	100
6	150
8	200
10	250

Each delay number denotes a 25-msec delay

FIGURE 7. - White Pine full-column round.



DELAY NUMBER	TIME INTERVAL, msec
0	—
2	50
4	100
6	150
8	200

Each delay number denotes a 25-msec delay

FIGURE 8. - White Pine parting shale round.

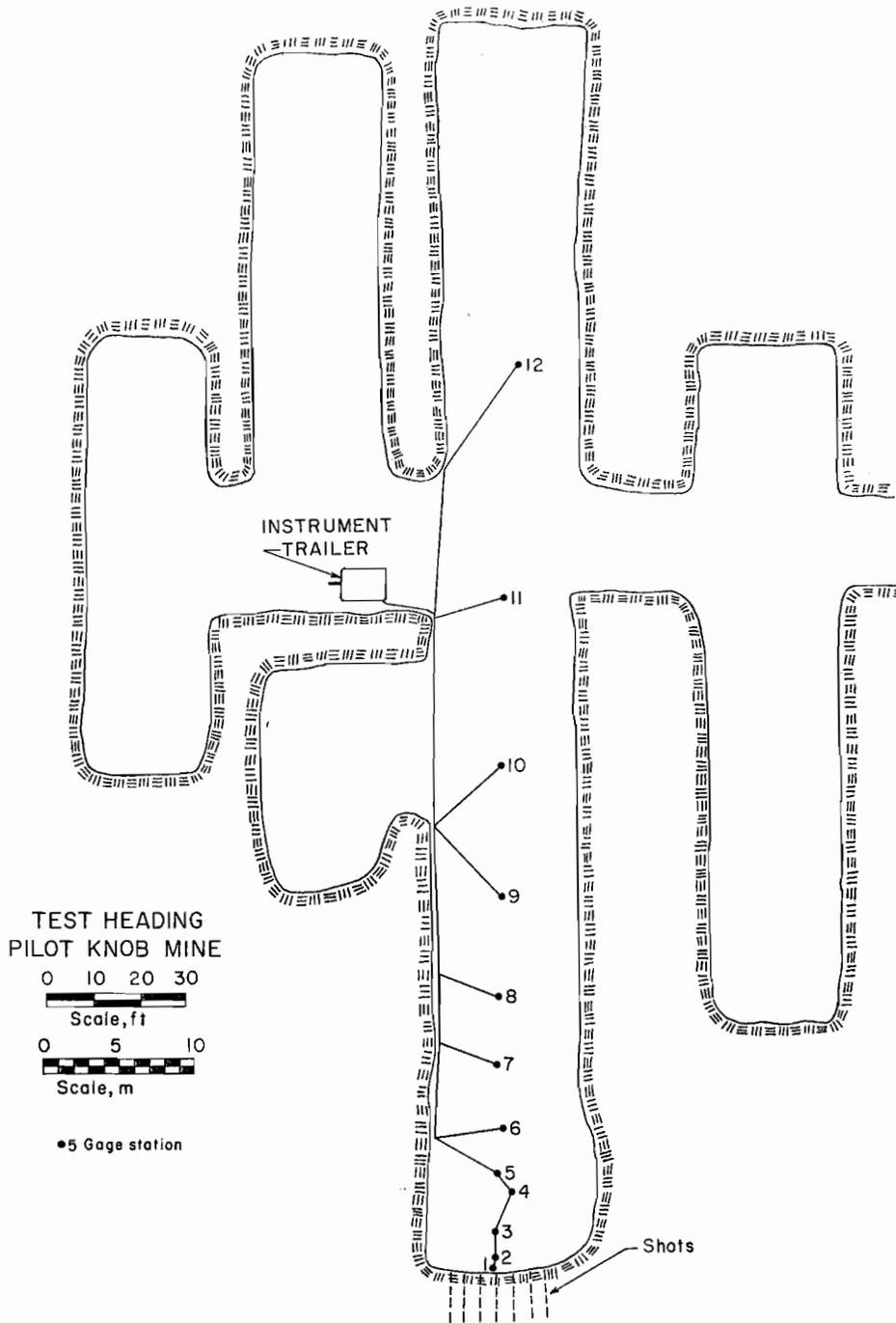


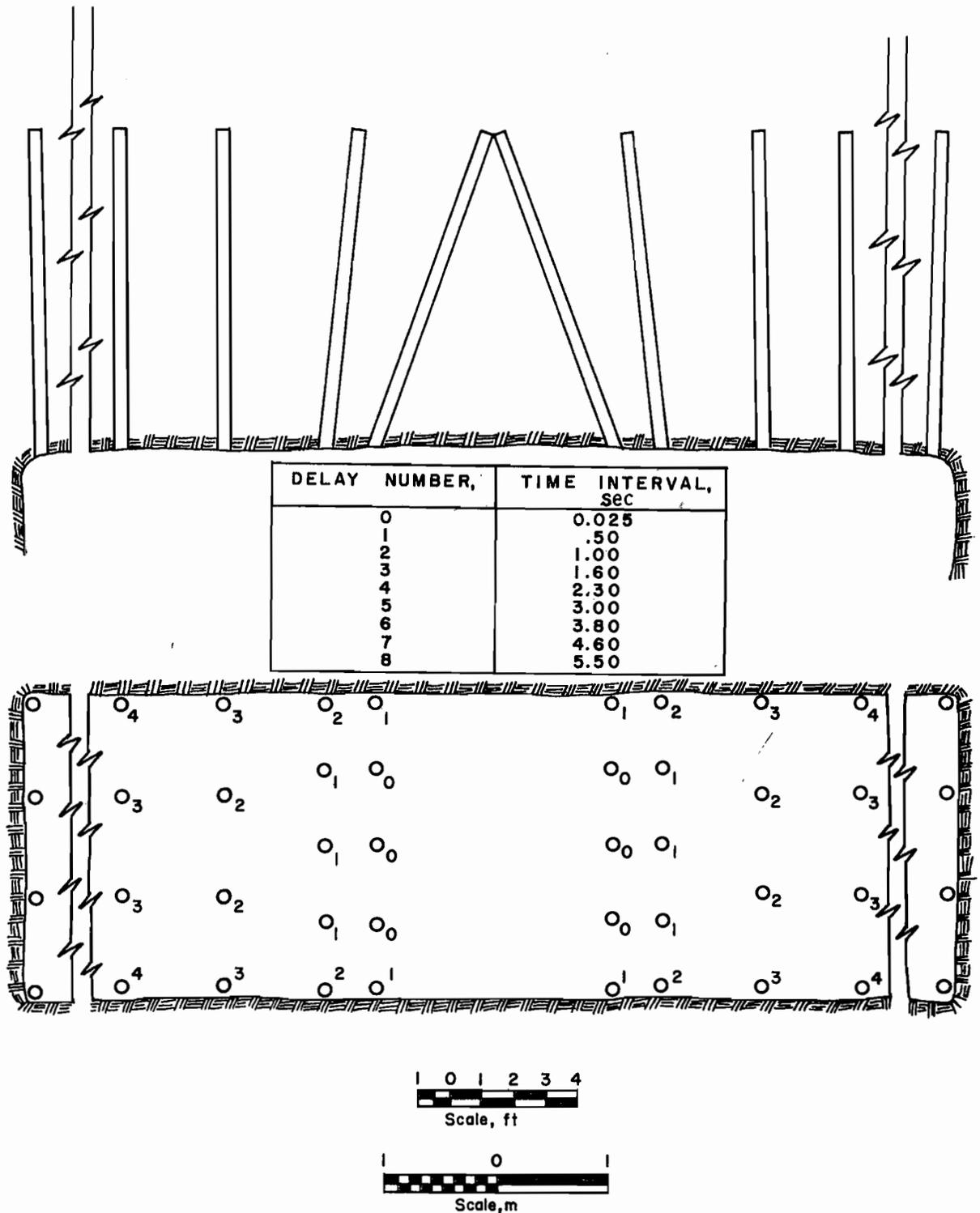
FIGURE 9. - Pilot Knob test heading.

acceleration pick-ups mounted, were then fastened to the roof using 3/8- by 3-in expansion bolts.

consisted of a 1-1/2- by 12-in (38 by 305 mm) cartridge of Gelex 2<sup>4</sup> (45 pct semigelatin) primed with a half-second-delay electric cap, followed by five 1-1/2- by 16-in (38 by 406 mm) cartridges of 50 pct low-density ammonia dynamite. Figure 11 shows relative locations of the gage stations, 1-stick shots, and production rounds. Maximum charge weight in the zero-delay period of the production rounds varied slightly (table 4).

The direct epoxy method of gage mounting used at White Pine could not be used at Shullsburg because of excessive moisture in the roof. An alternate method used 4-in-diam by 1/2-in-thick aluminum plates drilled and tapped for mounting both a velocity gage and an accelerometer. The plates, with velocity and

<sup>4</sup>Reference to specific brand names is made for identification only and does not imply endorsement by the Bureau of Mines.



(Face width and height vary among shots)  
 FIGURE 10. - Typical Shullsburg production round.

TABLE 4. - Shullsburg blast data

Shot	Type of explosive charge	Charge size weight	
		Lb	Kg
1-8.....	Semigelatin dynamite.....	0.92	0.42
9.....	Ammonia dynamite.....	37.2	16.9
10.....	.....do.....	35.5	16.1
11-16.....	.....do.....	41.6	18.9
17.....	.....do.....	36.8	16.7

## NORAD

A typical blast round in the exploratory drift at NORAD consisted of from 41 to 47 blastholes with a burn cut as shown in figure 12. Blastholes were 7 to 8 ft (2.1 to 2.4 m) deep and fired electrically in a delay sequence as indicated. Explosives used included various combinations of Gelamite 2 (45 pct semigelatin), Hercon 2 (50 pct low-density ammonia dynamite), and air-emplaced AN-FO. Velocity gages were mounted on aluminum plates as in the Shullsburg study. However, the gages were mounted along the walls of the excavation rather than the roof. Configuration of the shots and gage arrays are given in figure 13, and shot information, in table 5.

TABLE 5. - NORAD blast data

Shot	Type of explosive charge	Charge size weight	
		Lb	Kg
1-2.....	Semigelatin dynamite.....	6.8	3.1
3.....	.....do.....	36.6	16.6
4.....	.....do.....	7.5	3.4
5.....	.....do.....	3.7	1.7
6.....	.....do.....	11.2	5.1
7.....	Ammonia dynamite.....	3.4	1.5
8.....	.....do.....	15.1	6.8
9.....	.....do.....	19.7	9.0
10.....	.....do.....	18.7	8.5
11.....	.....do.....	47.0	21.4
12.....	Semigelatin dynamite.....	4.1	1.9
13.....	{ Semigelatin dynamite (17.7 lb, 7.6 kg).... Ammonia dynamite (56.7 lb, 25.7 kg)..... }	73.4	33.3

Conversion of Acceleration to Velocity

Several justifications exist for the measurement of particle acceleration despite the widely accepted use of particle velocity damage criteria and aside from energy considerations (4, 11). Determination of propagation equations requires a wide range of charge weights and shot-to-gage distances. Because of practical limitations on the upper values of these parameters, some very small shots and close-in measurements are required, and the resulting high-frequency, large-amplitude vibrations are better resolved using

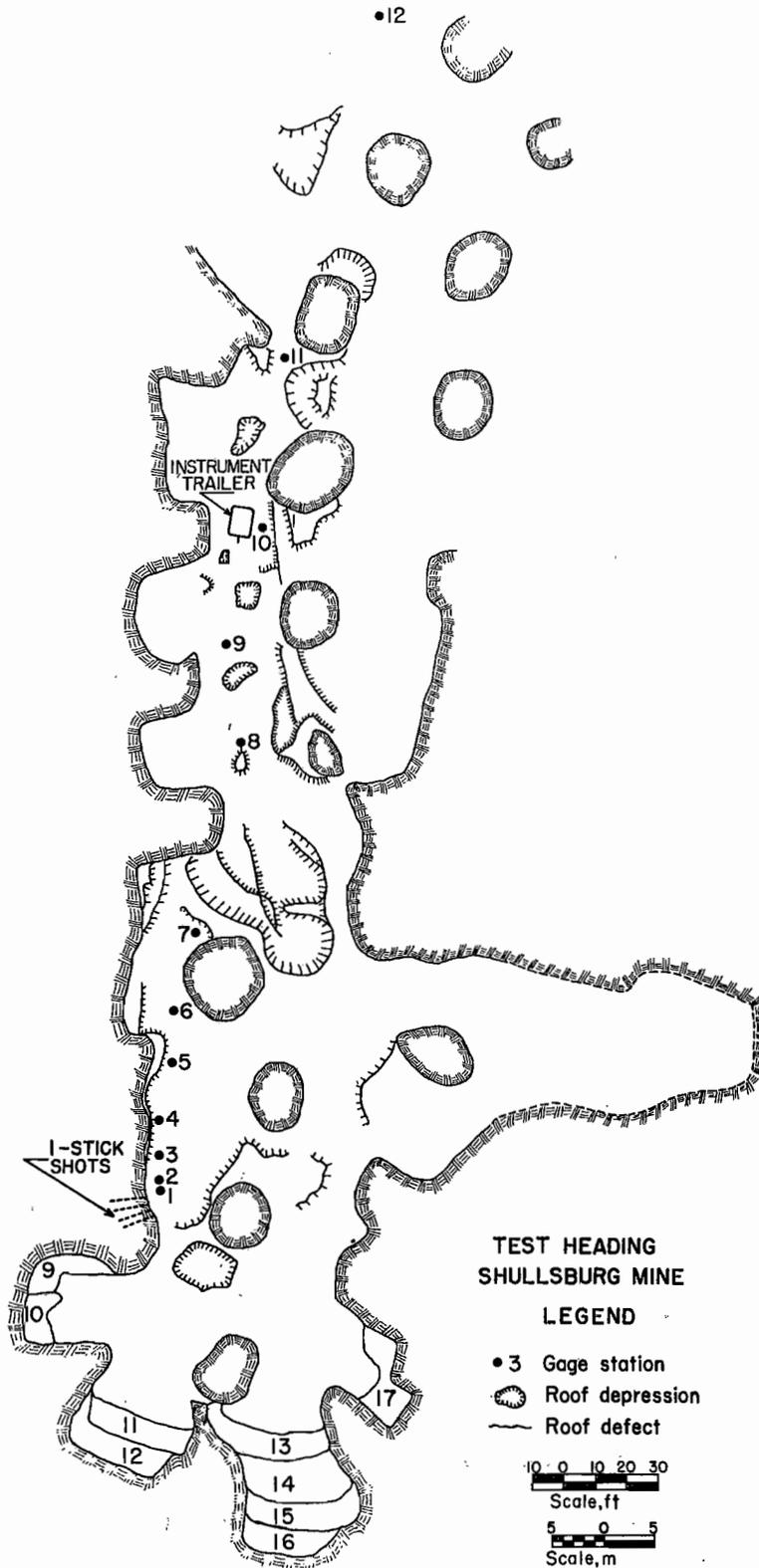


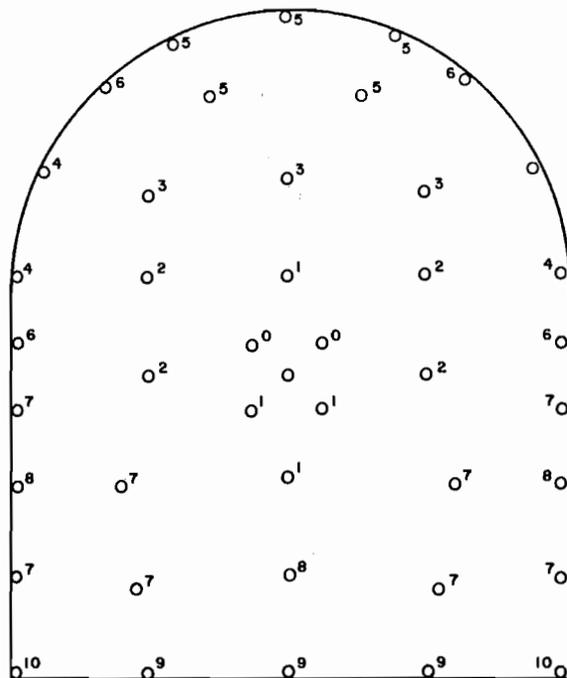
FIGURE 11. - Map of Shullsburg test heading.

appropriate frequency response of shock insensitive accelerometers. Analysis of the data in terms of particle velocity is not impaired, however, since the quantities of ground motion (acceleration, velocity, and displacement) are related through the operations of differentiation and integration as follows:

$$v = du/dt = \int a dt, \quad (3)$$

where u is displacement, v is velocity, a is acceleration, and t is time.

Blair and Duvall (1) showed that the relationship of equation 3 may be used, within the accuracy of the original measurements, to derive particle velocity from either acceleration or displacement records. Good agreement was obtained, using graphical methods, between derived and directly recorded quantities of ground motion. This indicates that records obtained using an appropriate transducer are true representations of ground motion and that propagation laws for peak amplitudes of displacement, velocity, or acceleration are independent of the type of gage used. Therefore, selected acceleration records from the White Pine and Pilot Knob investigations were integrated to determine propagation equations relating particle velocities to scaled distances. This allows a direct comparison of the



0' 1 2  
Scale, ft

0 1  
Scale, m

DELAY NUMBER	AVERAGE DELAY TIME, sec
0	0.008
1	.025
2	.30
3	.50
4	.75
5	1.00
6	1.50
7	2.00
8	2.50
9	3.00
10	3.50

FIGURE 12. - Typical NORAD blastround.

analog-to-digital transcription instrumentation to simplify the technique.

propagation characteristics of the different sites in terms of particle velocity and also places the underground data in compatible form for comparison with the quarry blast results. Three methods of conversion were investigated.

#### Numerical Methods

The mathematically correct process of performing integration or differentiation of nonperiodic time functions such as vibration records entails the use of Fourier techniques and high-speed digital computers. Nicholls, Johnson, and Duvall (4) used these techniques in their investigations to determine displacements and acceleration and to examine the relationship between instantaneous and delayed blasts. However, they suggested that although many operations can be performed more easily in the frequency domain, the use of Fourier analysis may not be justified if the only purpose is to determine frequency content of the signal.

A simpler numerical method was attempted on some of the Pilot Knob acceleration data. The records were digitized, input to a computer and integrated using Simpson's rule (9). This is essentially a computer extension of the graphical method of summing discrete differential areas under the time-series curve to obtain the integral. Good reproduction of waveforms was obtained; however, problems with base line shift, subjective errors, and the tedium of digitizing records resulted in abandonment of this method. Future work in this area requiring digital analysis should consider recording directly in digital format or using

#### Electronic Integration

The Pilot Knob acceleration recordings were electronically integrated using a time constant (RC) circuit unit made for use with the Burr-Brown preamplifiers. The electronic system consisted of a tape playback at 1/32

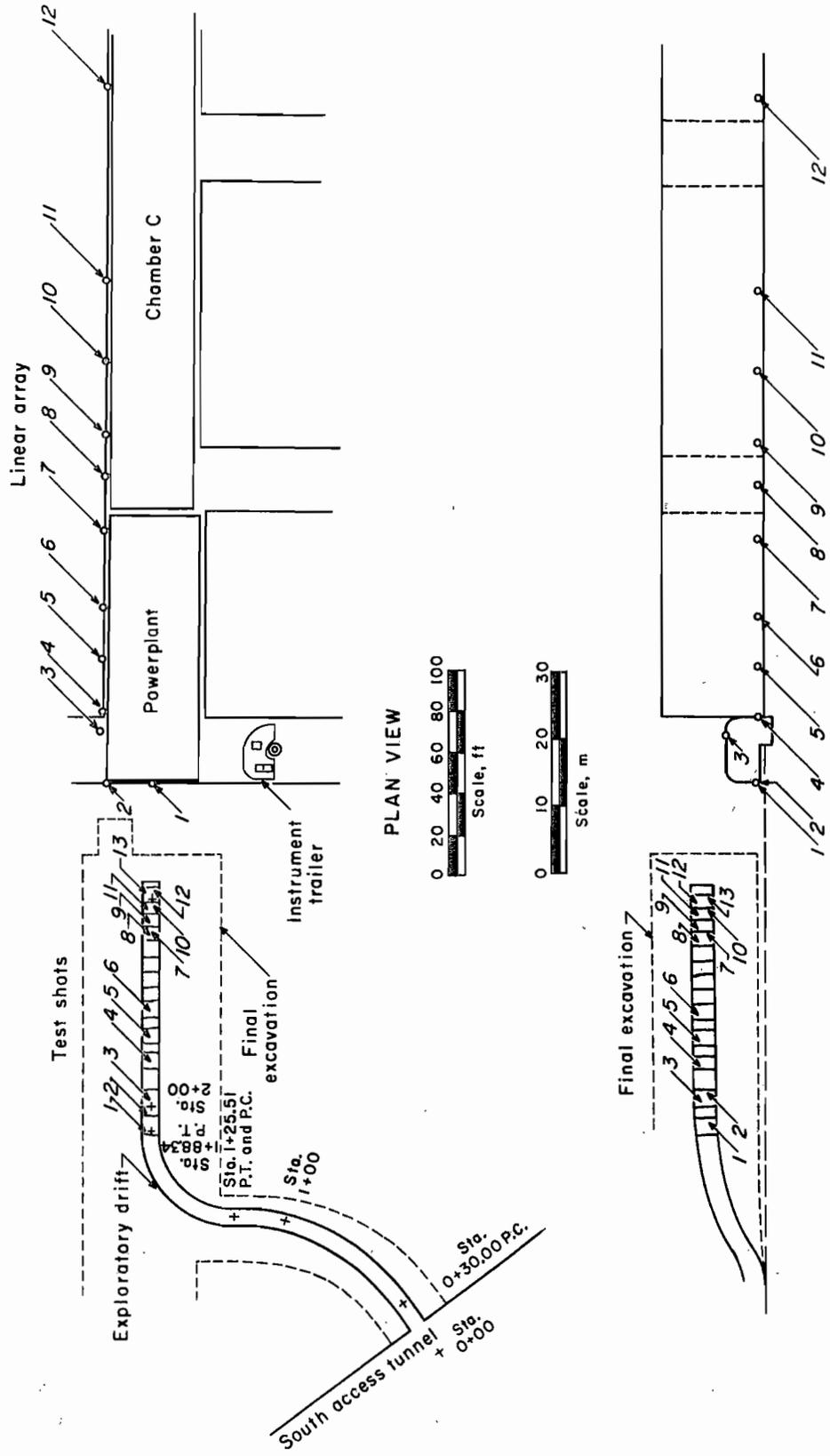


FIGURE 13. - Plan and profile views of NORAD exploratory drift.

of the original recording speed, preamplifier, integrator, amplifier, band-pass filter, and oscillograph. Theory required (8) that the RC integrating network have a time constant that is very large with respect to the longest period of the input signal, to avoid distortion. At the same time, the resulting approximation given by equation 4,

$$e_o \approx \frac{1}{RC} \int_0^t e_i dt, \quad (4)$$

shows the inverse relationship between the time constant, RC, and the output voltage,  $e_o$ , making the output very small for large values of RC. A compromise RC of 0.1 was used in the acceleration signal integration. Comparisons were made using the more ideal value of 1.0; however, this large time constant resulted in both very small signals for many of the records and problems with signal and noise separation. The 0.1 RC gave particle velocity traces similar to those obtained with the higher value and also in agreement with the digitally integrated results.

Two schemes were used to calibrate the electronic integration system. An experimental calibration was made by integrating several of the standard (20g peak-to-peak, 1,088 Hz) sinusoidal calibration signals originally recorded with each set of blasts at Pilot Knob. A constant was derived that accounted for the changes in the signal amplitudes due to the integrating, filtering, and two amplifying operations. Peak particle velocities were then calculated from simple measurements of the signal trace amplitudes on the processed record.

A theoretical calibration was made using equation 4, with an RC of 0.1, an input voltage,  $e_i$ , of  $\sin 2\pi ft$ , and the known amplifier settings. Again, a multiplying constant was derived, the value of which was within a few percent of the experimentally derived value.

A total of 73 acceleration records from the Pilot Knob study were electronically integrated to give particle velocity traces, representing the entire range of explosive sizes and three explosive types.

#### Simple Harmonic Motion

The most expedient method for deriving peak velocity magnitudes from acceleration records involves the approximation of ground motion by a simple sine or cosine function of the form:

$$a = k \sin 2\pi ft, \quad (5)$$

where  $a$  is acceleration,  $k$  is an amplitude constant,  $f$  is frequency, and  $t$  is time. Using this relationship and equation 3, velocity becomes

$$v = \int a = - (k/2\pi f) \cos 2\pi ft, \quad (6)$$

and peak amplitude of particle velocity is inversely proportional to the frequency. Peak velocity may then be calculated directly using the peak

acceleration and the estimated frequency of vibration at that portion of the record where the measurement is taken.

Nicholls and Fogelson (3) suggested that particle velocities calculated from displacement or acceleration data assuming simple harmonic motion would generally be less than particle velocities recorded directly. Therefore, an analysis was made to determine if the method could be used reliably for the underground data. For the Shullsburg site, where both particle velocities and accelerations were recorded concurrently, equation 6 was used to convert acceleration values to velocities from which an empirical propagation equation was derived. Figure 14 shows a comparison of the least-squares regression lines fitted to the square-root-scaled data for the directly recorded and derived velocity values. Some variation was expected since the number of acceleration data points was significantly less than the number of velocity measurements; however, the results exhibit reasonably good agreement, the derived velocity levels being slightly less than the direct measurements.

Additionally, regression lines from the Pilot Knob data, derived using equation 6, and a portion of the same data using electronic integration are compared in figure 15. Agreement is also good between these two methods with slight deviation expected because of variance in the number of data points analyzed.

#### Scaling Factors

The value of charge weight as a scaling factor for vibrations lies in the ability to predict maximum amplitudes from a wide range of charge weights. Nicholls (2) used cube root scaling and a series of small blasts with chemical explosives to reliably extrapolate predicted vibration amplitudes for a large nuclear detonation. Cube root scaling is supported by dimensional analysis if a spherical charge of constant density and increasing radius is assumed, resulting in a weight (volume) change that is proportional to the cube of the radius. Justification for square root scaling for most blasting situations arises from the procedures used to vary charge weights. Since charges are generally cylindrical and of fixed length, an increase in hole diameter results in a weight (volume) increase proportional to the square of the radius. This indicates square root scaling may be more appropriate and is supported by statistical analysis of data from 39 blasts ranging in size from 25 to 4,620 lb (11.4 to 200 kg) per delay period (4, p. 41). Square root scaling was also applicable at the White Pine and Pilot Knob sites. Cube root scaling was indicated at Shullsburg and NORAD; however, table 6 indicates that the differences are not that great. Correlation coefficients of the least-square regression lines through the scaled velocity data for both cube root and square root scaling are given for each site.

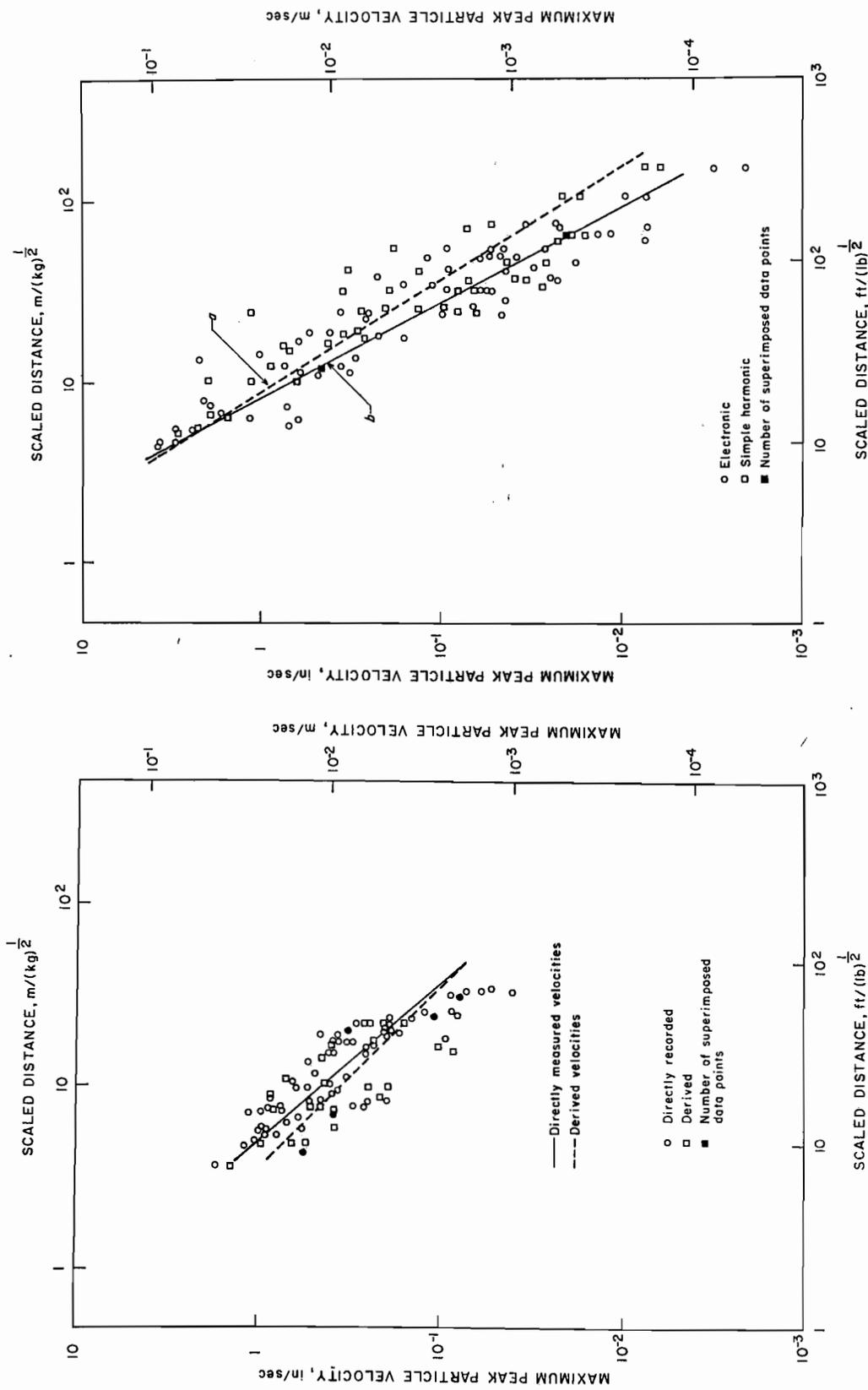


FIGURE 14. - Comparison of regression lines representing directly recorded and derived velocities.

FIGURE 15. - Least-squares regression lines of Pilot Knob data after (a) simple harmonic conversion and (b) electronic integration.

TABLE 6. - Correlation coefficients of least-squares regression lines for square root and cube root scaling

Test site	Scaling factor	Correlation coefficient
White Pine.....	0.50	0.88
	.33	.87
Shullsburg.....	.50	.86
	.33	.93
NORAD.....	.50	.91
	.33	.93
Pilot Knob.....	.50	.93
	.33	.90

Propagation Equations

The empirical propagation equations, as determined in the underground studies may be written in the general form:

$$v = K_1 (D/W^b)^{-n}, \tag{7}$$

or

$$aW^b = K_2 (D/W^b)^{-m}, \tag{8}$$

where  $v$  and  $a$  are particle velocity and acceleration, respectively, the unknown coefficients ( $K_1, K_2$ ) are the log-intercepts at unit scaled distance,  $b$  is the scaling exponent on the charge weight,  $W$ , and  $n$  and  $m$  are the decay exponents. If the quantity  $W^b$  is a scaling factor ( $b = 1/2$  or  $1/3$ ) then a plot of particle velocity versus scaled distance,  $D/W^b$ , (or scaled acceleration versus scaled distance) on log-log paper allows determination of the unknown site constants. Table 7 lists the values of these constants for the underground sites. By substituting the values into the general equations 7-8, the propagation laws for a particular site may be obtained. Comparisons of the plotted empirical equations for the underground sites are given in figures 16-18. The vertical lines represent  $\pm$  one standard deviation of the data. Figures 19-22 are included to indicate the number of points used to derive regression lines for the square-root-scaled velocity data.

TABLE 7. - Values of site constants for square root and cube root scaling

Site	b	$K_1$	n	$K_2$	m
White Pine.....	1/2	104	1.55	}25,000	2.21
	1/3	437	1.85		
Shullsburg.....	1/2	24	1.23	}14,000	2.07
	1/3	90	1.53		
NORAD.....	1/2	61	1.70	}	-
	1/3	560	2.04		
Pilot Knob.....	1/2	228	1.90	}82,000	2.65
	1/3	249	1.93		

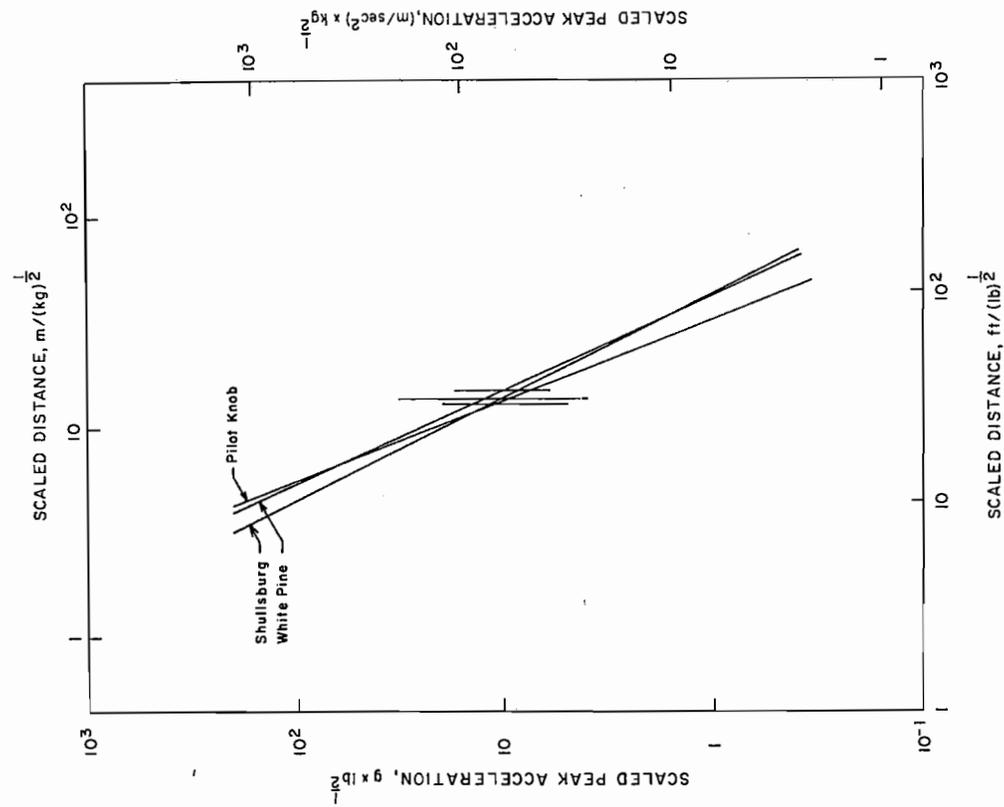


FIGURE 16. - Least-squares regression lines of square-root-scaled velocity data.

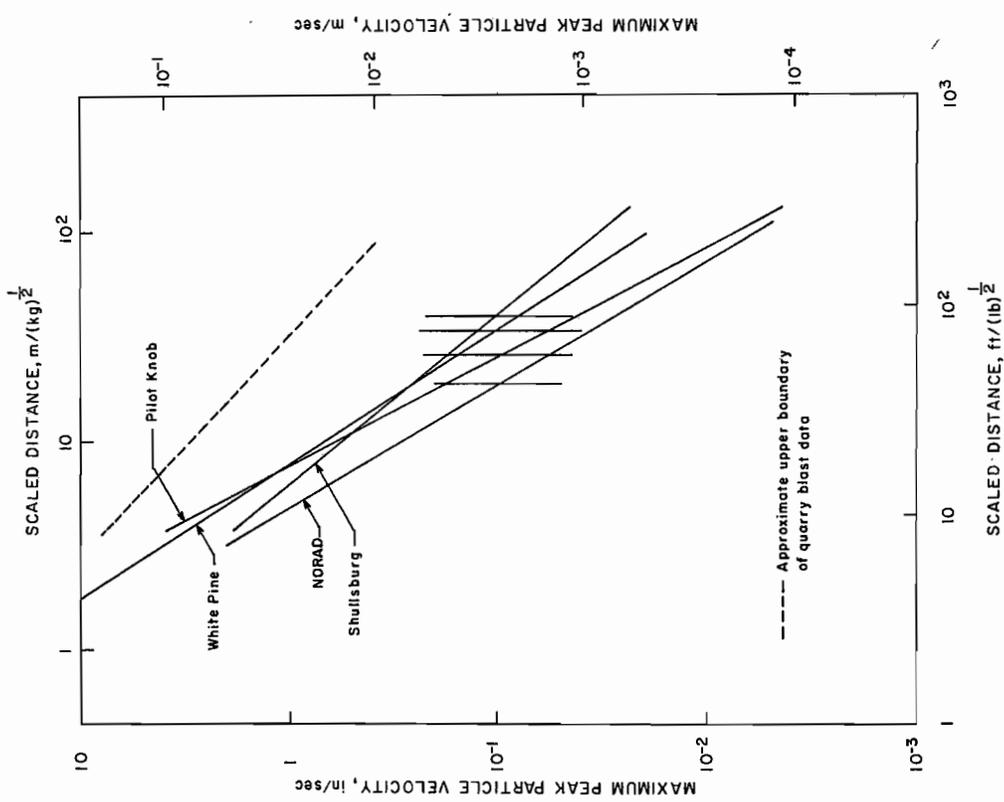


FIGURE 17. - Least-squares regression lines of square-root-scaled acceleration data.

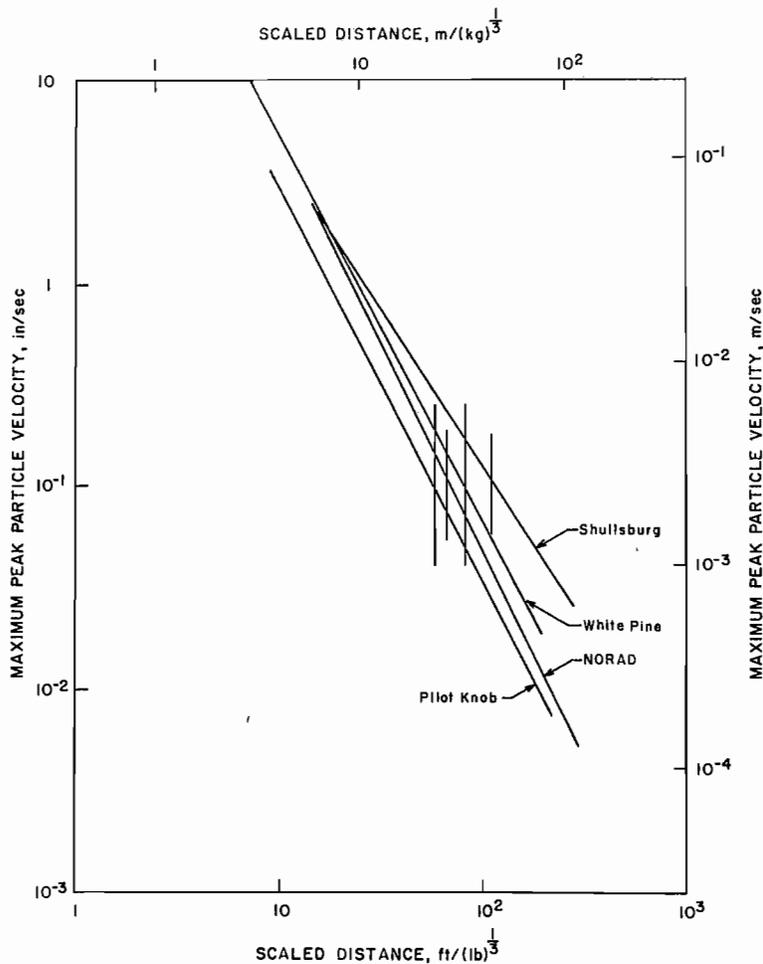


FIGURE 18. - Least-squares regression lines of cube-root-scaled velocity data.

appropriate, whereas dimensional analysis supports the cube root factor. Only one study, Pilot Knob, involved the statistical determination of a best fit scaling factor for the range of charge weights used, that being 0.55. Differences in grouping of scaled data, as measured by the correlation coefficients of the least-squares regressions (table 7), are not significant in estimating vibration levels.

Regression lines through the square-root-scaled velocity data from the underground sites are given in figure 16. The dashed line represents the upper limit envelope of the quarry blast data. A few points from the White Pine study plotted above this line (fig. 19). With this exception, however, the bandwidth of the square-root-scaled underground data is in good agreement with the quarry data. At a scaled distance of 50 ft/lb<sup>2</sup>, the vibration level is considerably less than 2.0 in/sec peak particle velocity indicating that the underground data appear to follow a propagation law similar to that determined from quarry blasting. It should be noted that the damage criterion of 2.0 in/sec applies to residential structures and quarry blasting; no attempt

## DISCUSSION OF RESULTS

The vibration measurements from underground blasting represent a wide range of rock types (table 1) and mining configurations. The effect of these site parameters on the log-intercepts and slopes of the regression lines through the scaled data for each site is evident from table 7 and figures 16-18. No analysis was made to determine if the data from the four sites could be pooled statistically to obtain a general propagation equation. Comparison of the equations suggests, however, that the differences are not as great as might have been expected, and that the data might be combined if a proper scaling exponent for the charge weight were established. Three of the underground studies investigated square root versus cube root scaling as alternatives because the quarry studies (4) had shown the square root scaling factor to be more appro-

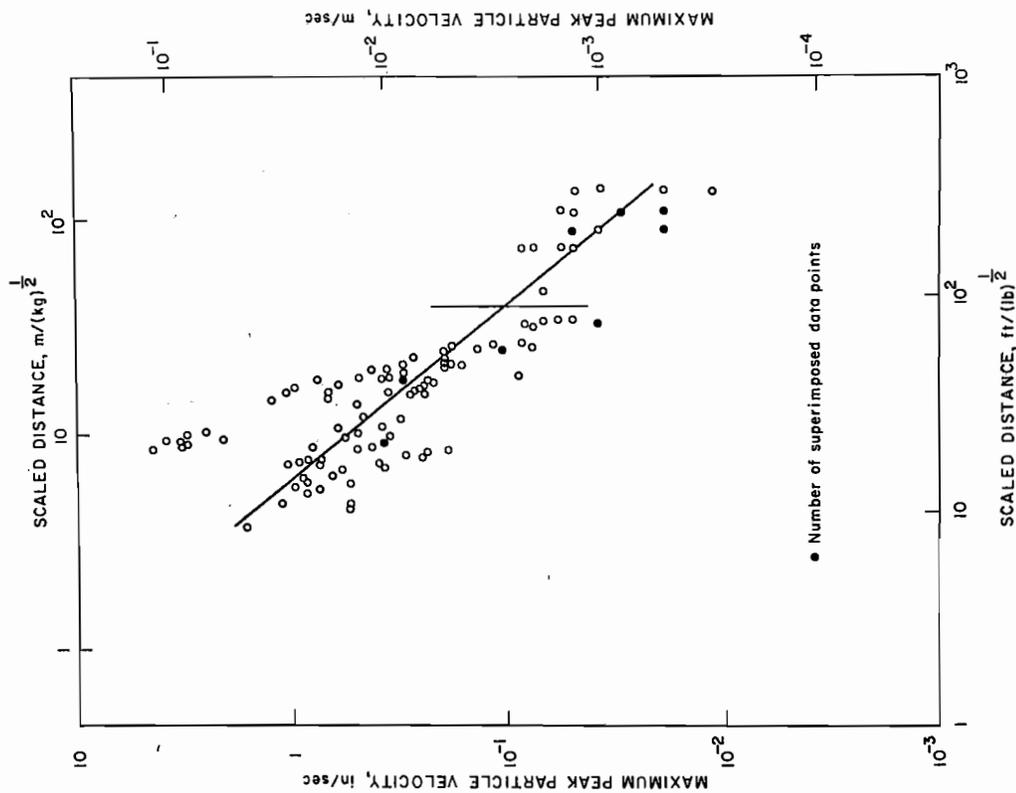


FIGURE 20. - Shullsburg velocity data.

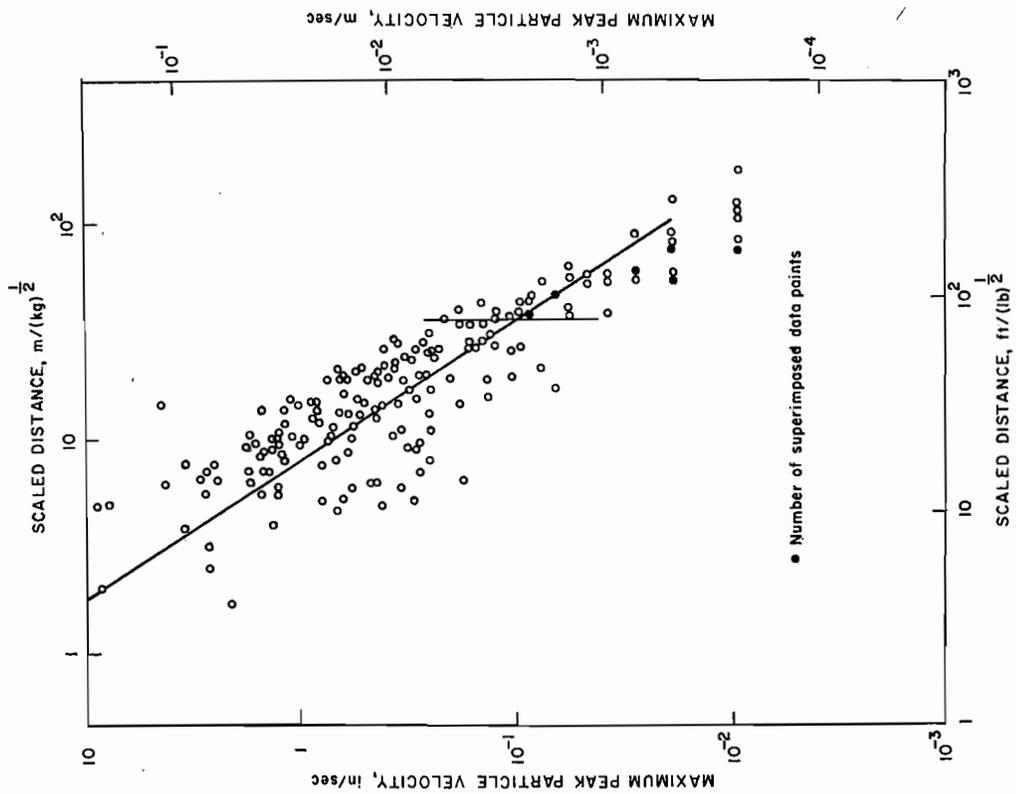


FIGURE 19. - White Pine velocity data.

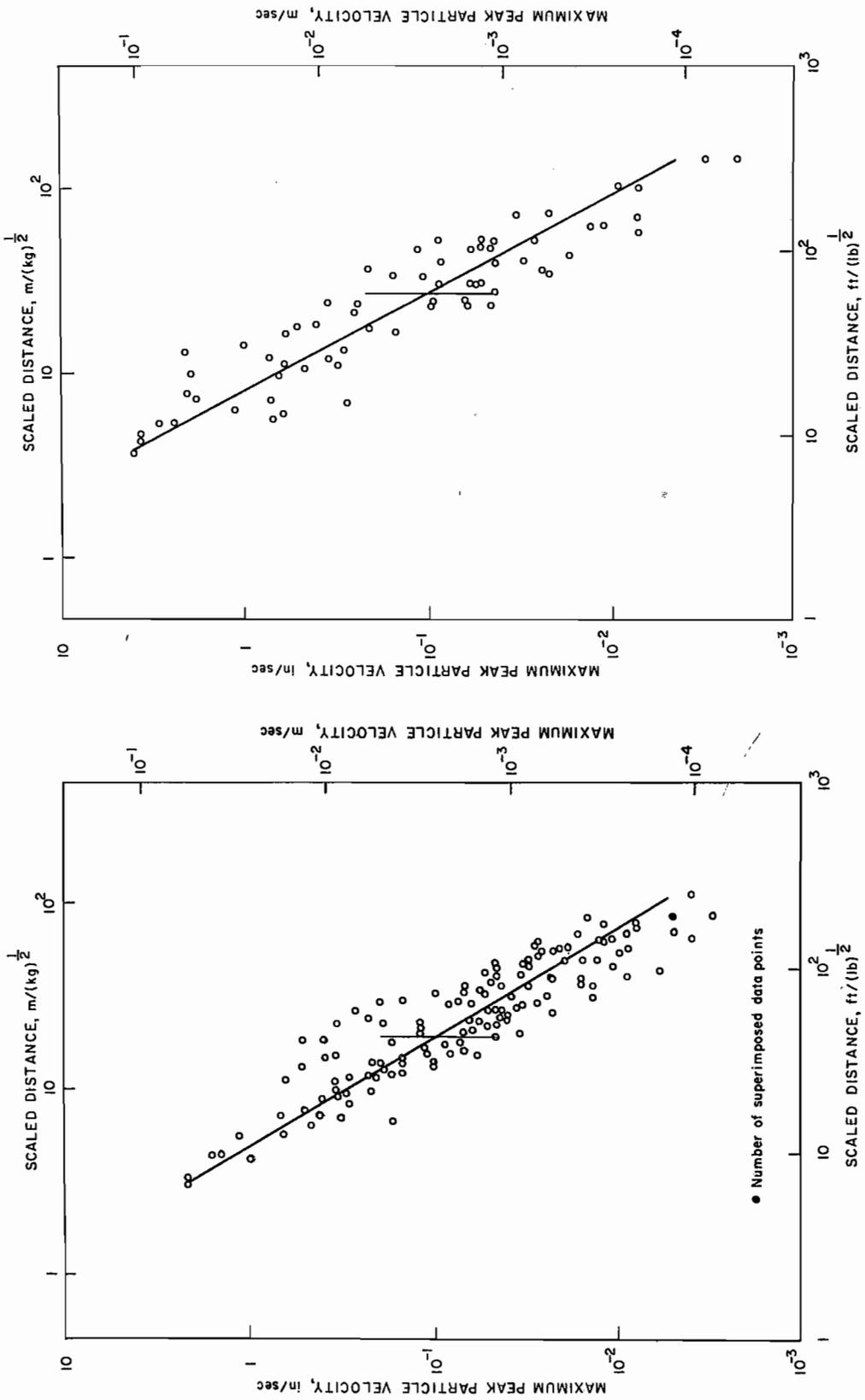


FIGURE 21. - NORAD velocity data.

FIGURE 22. - Pilot Knob velocity data.

was made to correlate blast vibrations with damage to mine support structures. Further research, including consideration of the particular mining method, will be required to determine permissible levels of vibration regarding damage to underground structures.

The graphs of figures 16-17 provide a convenient means for predicting vibration amplitudes over a wide range of conditions. If particle velocity is to be measured directly, figure 16 may be used to estimate initial instrumentation settings. If accelerometers are to be used, figure 17 provides a first approximation to vibration levels that may be expected. Particle accelerations may then be converted to velocities using one of the methods discussed in a previous section and a propagation law derived for the site.

Typical underground blast rounds utilize either long period or millisecond-delay blasting caps to allow displacement of broken rock between detonations, preventing freezing of the round. The zero-delay portion is "tight" or has less room to move rock than later delays. It might be expected that proportionately more vibration energy is transmitted to the unbroken rock than in later delays. The White Pine and Shullsburg studies indicated that the maximum vibration amplitudes were produced by the zero delay and that they could be effectively reduced by decreasing the amount of explosive in the zero-delay period.

Additionally, the White Pine and Pilot Knob studies showed that vibration levels can be significantly reduced by using AN-FO, where practical. Maximum amplitudes of vibration from AN-FO blasts were less than those from equivalent dynamite or slurry shots.

Different situations in subsurface blasting will dictate the permissible levels of vibration. If a mine is excavated in competent rock, remote from manmade structures, there may be no concern. However, excavations in unstable ground where vibrations may contribute to hazardous conditions, or operations near residential structures should benefit from the results of these underground investigations.

#### CONCLUSIONS

The square-root-scaled regression data of figures 16-17 may be used for practical engineering predictions of maximum particle velocity or acceleration amplitudes from subsurface blasting.

For square root scaling, the subsurface vibration data appear to follow a propagation law similar to that determined from quarry blasting. The vibration level from tunnel blasts measured underground is less than 2 in/sec at a scaled distance of 50.

The requirement to record particle accelerations poses no problem in analyzing vibration data since the assumption of simple harmonic motion provides a convenient method of obtaining reasonable approximation to particle velocities. If waveform is to be preserved, however, either electronic conversion or digital processing may be required.

## REFERENCES

1. Blair, B. E., and W. I. Duvall. Evaluation of Gages for Measuring Displacement, Velocity, and Acceleration of Seismic Pulses. BuMines RI 5073, 1954, 21 pp.
2. Nicholls, H. R. A Case Study of the Validity of Scaling Laws for Explosion-Generated Motion. BuMines RI 6472, 1964, 14 pp.
3. Nicholls, H. R., and D. E. Fogelson. Controlling Seismic Effects of Blasting. Nat. Safety Cong. Trans., 1967, pp. 46-53.
4. Nicholls, H. R., C. F. Johnson, and W. I. Duvall. Blasting Vibrations and Their Effects on Structures. BuMines Bull. 656, 1971, 105 pp.
5. Olson, J. J., R. A. Dick, J. L. Condon, and D. E. Fogelson. Mine Roof Vibrations From Underground Blasts. BuMines RI 7330, 1970, 55 pp.
6. Olson, J. J., R. A. Dick, D. E. Fogelson, and L. R. Fletcher. Mine Roof Vibrations From Production Blasts, Shullsburg Mine, Shullsburg, Wis. BuMines RI 7462, 1970, 35 pp.
7. Olson, J. J., D. E. Fogelson, R. A. Dick, and A. D. Hendrickson. Ground Vibrations From Tunnel Blasting in Granite: Cheyenne Mountain (NORAD), Colo. BuMines RI 7653, 1972, 25 pp.
8. Ryder, J. D. Engineering Electronics. McGraw-Hill Book Co., Inc., New York, 1957, 665 pp.
9. Scarborough, J. B. Numerical Mathematical Analysis. Johns Hopkins Press Baltimore, Md., 1962, 594 pp.
10. Scott, J. H., F. T. Lee, R. D. Carroll, and C. S. Robinson. The Relationship of Geophysical Parameters to Engineering and Construction Parameters in the Straight Creek Tunnel Pilot Bore, Colo. Internat. J. Rock Mech. Min. Sci., v. 5, No. 1, 1968, pp. 1-30.
11. Siskind, D. E. Ground and Air Vibrations From Blasting. Subsection 11.8 in SME Mining Engineering Handbook, ed. by A. B. Cummins and I. A. Given. Society of Mining Engineers of the American Institute of Mining, Metallurgical, and Petroleum Engineers, Inc., New York, 1973, pp. 11-99--11-111.
12. Siskind, D. E., J. J. Snodgrass, R. A. Dick, and J. N. Quiring. Mine Roof Vibrations From Underground Blasts, Pilot Knob, Mo. BuMines RI 7764, 1973, 21 pp.
13. Siskind, D. E., R. C. Steckley, and J. J. Olson. Fracturing in the Zone Around a Blasthole, White Pine, Mich. BuMines RI 7753, 1973, 20 pp.

